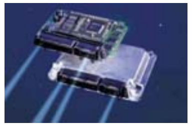
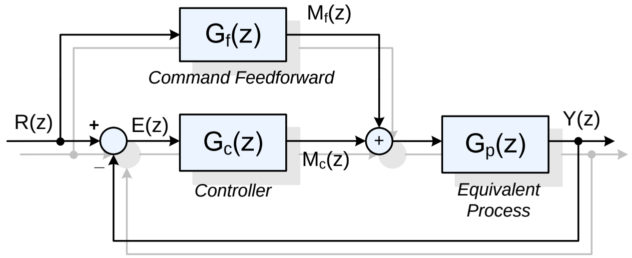
Outline – Controller Design III



- Command Feedforward
  - Example
- Zero Phase Error Tracking Controller (ZPETC)
  - Example
- Disturbance Decoupling
  - Example
- Cascade Control
  - Industry Standard Motion Controller



Command Feedforward Controller



Let us design a command feedforward controller $G_f(z)$ to improve the tracking accuracy of this system:

$$Y(z) = G_p(z)[G_c(R - Y) + G_f(z)R(z)]$$

Hence,

$$\frac{Y(z)}{R(z)} = \frac{G_p(z)(G_f + G_c)}{1 + G_p(z)G_c(z)}$$

CF Controller (Cont'd)

Suppose that Y(z) = R(z) is desired. In that case,

$$\frac{Y(z)}{R(z)} = \frac{G_p(z)(G_f + G_c)}{1 + G_p(z)G_c(z)} = 1$$

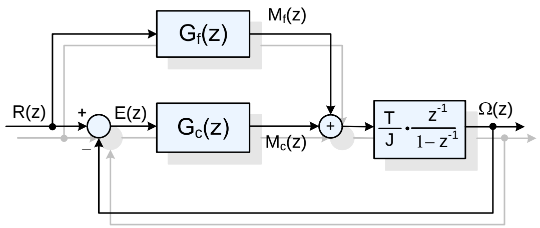
Hence,

$$G_f(z) = \frac{1}{G_p(z)}$$

Notice that this approach yields the transfer function of an *open-loop controller*. When $1/G_p(z)$ is unfeasible to implement, the closest realizable function is sought such as

$$G_f(z) = \frac{z^{-1}}{G_p(z)} \quad \text{or} \quad G_f(z) = \frac{z^{-2}}{G_p(z)}$$

Example



As an example, we shall develop a command feedforward controller (CFC) for the motor control system shown:

$$\frac{M_f(z)}{R(z)} = G_f(z) = \frac{1}{\hat{G}_p(z)} \Rightarrow \quad G_f(z) = \frac{\hat{J}}{T} \cdot \frac{1 - z^{-1}}{z^{-1}}$$

Notice that the corresponding difference equation is

$$m_f(k-1) = \frac{\hat{J}}{T}[r(k) - r(k-1)]$$

Solution (Cont'd)

Since we would like to obtain $m_f(k)$, the difference equation should be shifted forward in time:

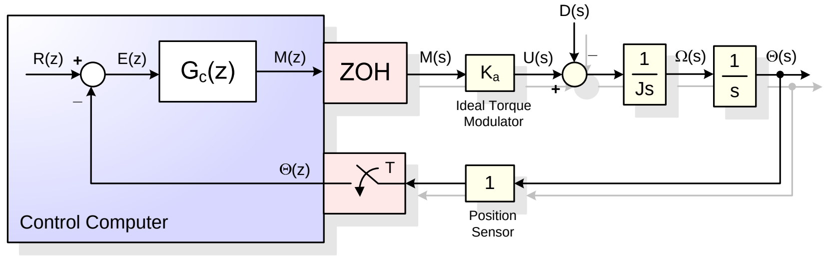
$$m_f(k) = \frac{\hat{J}}{T}[\underbrace{r(k+1)}_{future\ command} - r(k)]$$

Assuming that r(k+1) is not available, a relaxed CFC is to be devised:

$$G_f(z) = \frac{z^{-1}}{G_p(z)} = \frac{\hat{J}}{T} \cdot \frac{1-z^{-1}}{1}$$

This CFC does not require the future values of the command. However, this choice implies that y(k) = r(k-1). Please keep in mind that $G_f(z)$ itself must be stable for the sake of internal stability of the system.

Example



Consider the position motor control system with J = 0.001 [kgm^2] and K_a = 10 [Nm/V]. Design a CFC for this system.

Solution

With D(s) = 0; the discrete transfer function for the overall system can be expressed as

$$\frac{\Theta(z)}{M(z)} = G_p(z) \triangleq \frac{K_a T^2}{2J} \cdot \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^2}$$

Therefore:

$$\frac{M_f(z)}{R(z)} = G_f(z) = \frac{1}{\hat{G}_p(z)} \Rightarrow \quad G_f(z) = \frac{2\hat{J}}{K_a T^2} \cdot \frac{(1-z^{-1})^2}{\underbrace{(1+z^{-1})}_{forced\ osc.} \underbrace{z^{-1}}_{future\ value}}$$

With only the CFC in place, let us apply a ramp input to the system:

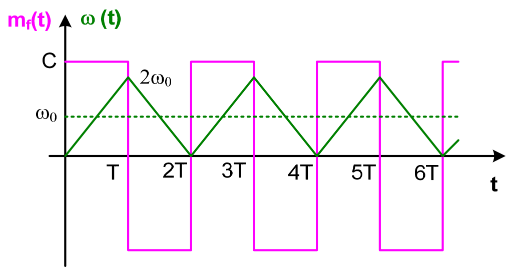
$$R(z) = \omega_0 \frac{Tz^{-1}}{(1-z^{-1})^2}$$

where ω_0 is the slope of the ramp corresponding to a constant speed.

Solution (Cont'd)

Consequently, the CFC output becomes

$$M_f(z) \triangleq \underbrace{\frac{2J\omega_0}{K_a T^2}}_{\triangleq C} \cdot \frac{1}{1+z^{-1}} = C\underbrace{(1-z^{-1}+z^{-2}-z^{-3}+\ldots)}_{indicates\ forced\ oscillations\ at\ (1/T)}$$



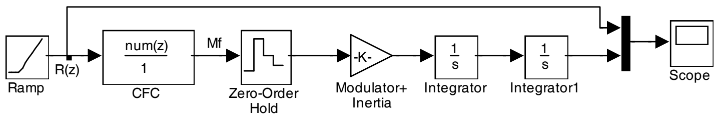
Solution (Cont'd)

Therefore, a more feasible CFC alternative would be

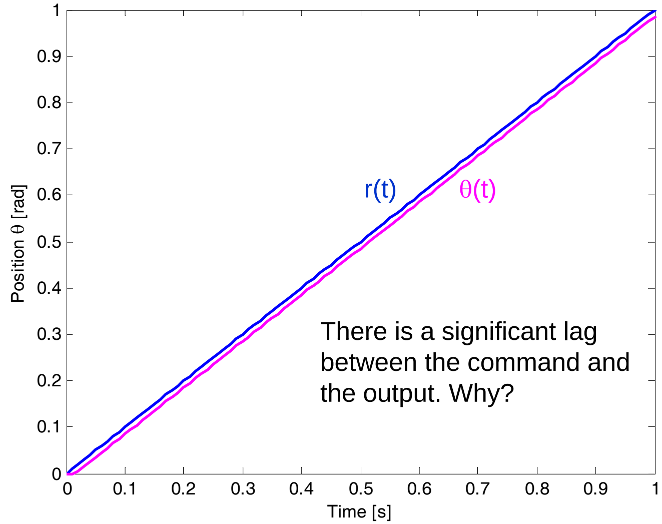
$$G_f(z) = \frac{\hat{J}}{K_a} \boxed{\frac{(1-z^{-1})^2}{T^2}}$$

Acceleration estimate via 2nd order backward difference method

In this case, $m_f(k)$ corresponds to the inertial torque command to the modulator which in turn yields the desired velocity and acceleration profiles. Let us simulate the system with this CFC and observe the tracking error.



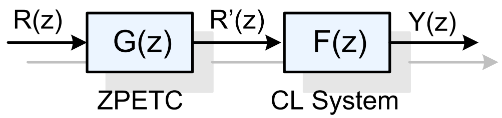
Solution (Cont'd)



Zero-Phase-Error Tracking Controller

- Another popular CFC is *zero-phase-error tracking controller* (ZPETC).
  - Proposed by Tomizuka in 1987,
  - Achieves zero phase error in command tracking,
  - Also applicable to non-minimum phase systems.
- Feasible inverse model is formed:
  - Effects of the CL poles and stable CL zeros are cancelled first,
  - Forward filter is included to eliminate the phase distortion introduced by the remaining (unstable) CL zeros.
- ZPETC requires the future values of command:
  - Feasible for most applications:
    - Robotics, Automation, and CNC Machine Tools

ZPETC (Cont'd)



Perfect tracking implies that Y(z) = R(z):

$$G(z^{-1}) = \frac{1}{F(z^{-1})} = \frac{A(z^{-1})}{B(z^{-1})}$$

When F(z) constitutes zeros residing outside the unit circle, such an approach leads to unstable G(z). To be specific, let $F(z^{-1})$ be

$$\frac{Y(z^{-1})}{R'(z^{-1})} = F(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{z^{-d}B^{+}(z^{-1})B^{-}(z^{-1})}{A(z^{-1})}$$

Here, $B^{+}(z^{-1})$ is a *factored* polynomial whose roots (i.e. zeros) are inside the unit circle while $B^{-}(z^{-1})$ is the one with its roots lying outside the circle.

ZPETC (Cont'd)

Tomizuka suggested a novel controller to attain zero phase error in command tracking:

$$G(z^{-1}) = \frac{A(z^{-1})B^-(z)}{z^{-d}B^+(z^{-1})[B^-(1)]^2}$$

Here $B^-(z)$ is the polynomial formed by substituting z into z^{-1} while $B^-(1)$ is the constant obtained by $B^-(z=1)$.

$$\therefore \quad \frac{Y(z^{-1})}{R(z^{-1})} = G(z^{-1})F(z^{-1}) = \frac{B^-(z^{-1})B^-(z)}{[B^-(1)]^2}$$

For sinusoidal commands, the phase shift introduced by $G(z^{-1})F(z^{-1})$ can be computed by simply replacing z^{-1} by $e^{-j\omega T}$ ($0 \le \omega \le \pi/T$):

$$\angle \frac{Y(j\omega)}{R(j\omega)} = \angle \frac{B^-(z^{-1})\big|_{z^{-1}=e^{-j\omega T}} B^-(z)\big|_{z=e^{j\omega T}}}{[B^-(1)]^2} = 0$$

Consequently, zero phase lag is obtained.

Example – ZPETC

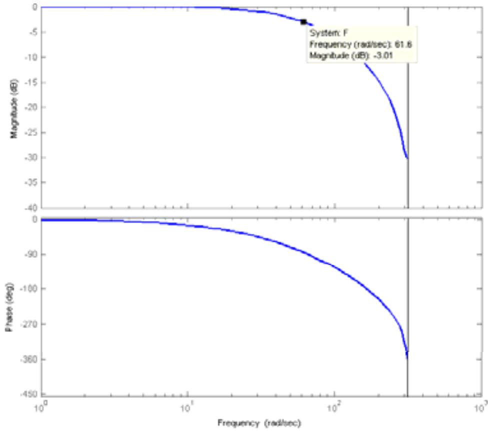
Consider a DTS with T = 0.01 s:

$$F(z) = \frac{0.2(z+1.25)}{(z-0.1)(z-0.5)}$$

That is,

$$\frac{Y(z^{-1})}{R'(z^{-1})} = \frac{z^{-1}(0.2+0.25z^{-1})}{1-0.6z^{-1}+0.05z^{-2}}$$

Bode Plot of Y(jω)/R'(jω):



Bandwidth is about 10 Hz. However, the command tracking performance (at even low frequencies) is unsatisfactory due to the phase lag introduced!

Example – ZPETC Design

To eliminate this phase lag, we shall design a ZPETC for this system:

$$A(z^{-1}) = 1-0.6z^{-1}+0.05z^{-2} \qquad B^-(z^{-1}) = 0.2+0.25z^{-1}$$

$$B^+(z^{-1}) = 1 \qquad B^-(z) = 0.2+0.25z; \quad B^-(1) = 0.45$$

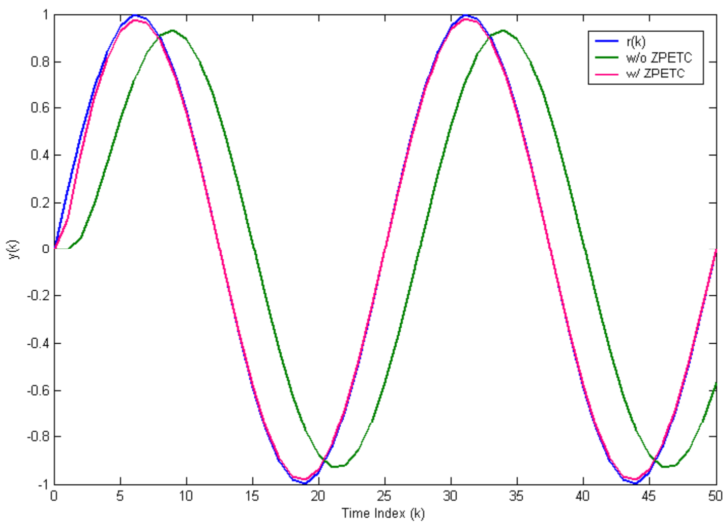
Hence, $$\frac{R'(z^{-1})}{R(z^{-1})} = G(z^{-1}) = \frac{(1-0.6z^{-1}+0.05z^{-2})(0.25+0.2z^{-1})}{z^{-2}(0.45)^2}$$

Corresponding CCDE becomes

$$r'(k) = \underbrace{1.2346\cdot r(k+2)+0.2469\cdot r(k+1)}_{\text{requires future command values}} - 0.5309\cdot r(k) + 0.0494\cdot r(k-1)$$

As an illustration, let $r(k) = \sin(\omega T k)$ where $\omega = 8\pi$ [rad/s].

Response to Sinusoidal Command

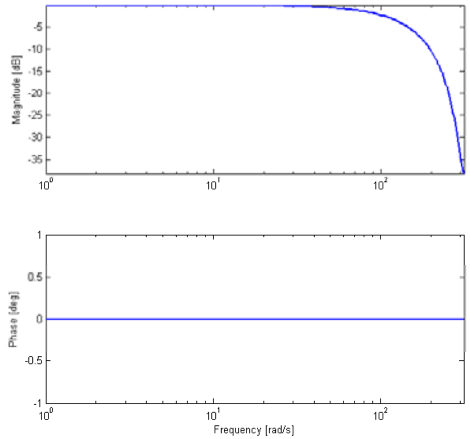


M-Script for Calculating Reponses

```
T = .01; A = [1 -.6 .05]; B = [0 .2 .25];
p = conv(A,[.25 .2])./(.45^2);
k = [0:50]'; r = sin(8*pi*k*T); rp = zeros(51,1);
%
% Compute the output of ZPETC r'(k)
%
for i = 0:50
  rp(i+1) = p*sin(8*pi*T*[i+2; i+1; i; i-1]);
end
%
% Calculate the responses to r(k) and r'(k)
%
y = filter(B,A,r); y0 = filter(B,A,rp);

plot(k,[r y y0]);
xlabel('Time Index (k)'); ylabel('y(k)');
legend('r(k)','w/o ZPETC','w/ ZPETC');
```


Bode Diagram w/ ZPETC

Bode Plot of Y(jω)/R(jω):



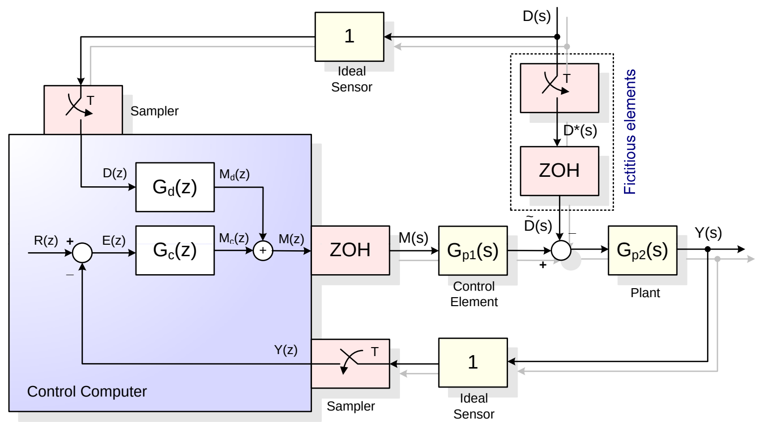
M-Script:

```
wT = linspace(.01,pi,100)';
M = zeros(100,1); P = M; T=.01;

H = (.2+.25*exp(-j*wT)).* ...
    (.2+.25*exp(j*wT))/(.45^2);
M = 20*log10(abs(H));
P = angle(H)*180/pi;

subplot(211)
  semilogx(wT/T,M);
  ylabel('Magnitude [dB]');
  axis('tight');
subplot(212);
  semilogx(wT/T,P);
  xlabel('Frequency [rad/s]');
  ylabel('Phase [deg]');
  axis('tight');
```


Disturbance Input Decoupling (DID) Method

- Under some circumstances, it is possible to measure the disturbance.
  - Ambient temperature in climate control system
  - Cross-wind speed in the auto-pilot system of a jet-liner.
- One can cancel the effect of disturbance using this measurement.

DID Method (Cont'd)

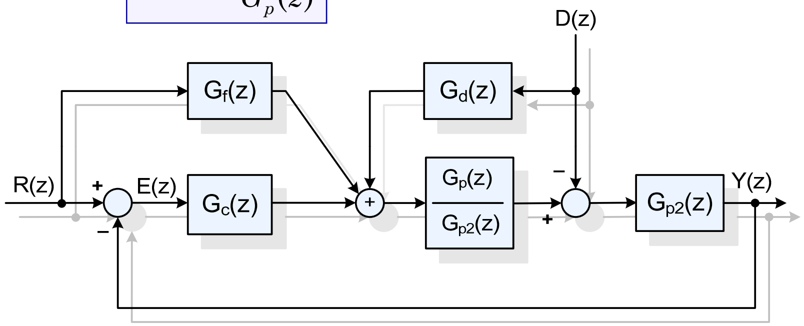


To decouple the disturbance input, the contribution of the DID controller should cancel the effect of the disturbance on the output.

DID Method (Cont'd)

$$D(z)G_d(z)\underbrace{Z\{G_{zoh}(s)G_{p1}(s)G_{p2}(s)\}}_{\triangleq G_p(z)} - D(z)\underbrace{Z\{G_{zoh}(s)G_{p2}(z)\}}_{\triangleq G_{p2}(z)} = 0$$

Then $$G_d(z) \triangleq \frac{G_{p2}(z)}{G_p(z)}$$



Back to Diagram

DID Method (Cont'd)

How do all these controllers affect the overall dynamics of this system? Let us try to address this question:

$$Y(z) = -D(z)G_{p2}(z) + D(z)G_d(z)\frac{G_p(z)}{G_{p2}(z)}G_{p2}(z)$$

$$+R(z)G_f(z)\frac{G_p(z)}{G_{p2}(z)}G_{p2}(z)$$

$$+(R-Y)G_c(z)\frac{G_p(z)}{G_{p2}(z)}G_{p2}(z)$$

Rearranging this equation yields

$$Y(z)[1+G_cG_p(z)] = D(z)[G_dG_p(z) - G_{p2}]$$

$$+R(z)[G_fG_p(z) + G_cG_p(z)]$$

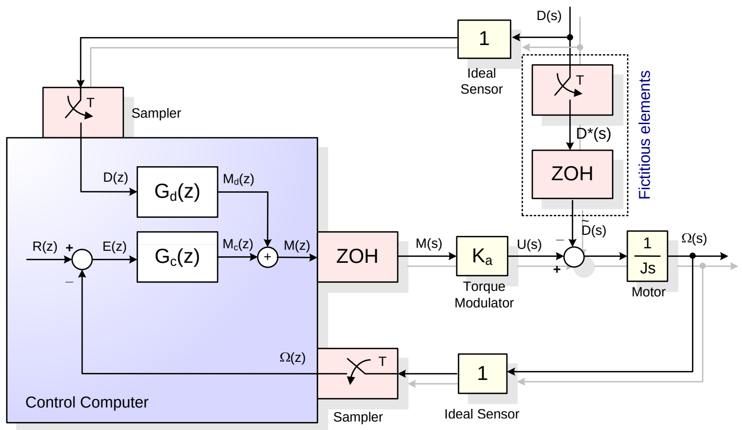
DID Method (Cont'd)

Substituting $G_f(z)$ and $G_d(z)$ into the expression gives

$$Y(z) = \frac{1}{1+G_cG_p(z)}\left\{D(z)\underbrace{\left[\frac{\hat{G}_{p2}(z)}{\hat{G}_p(z)}G_p(z) - G_{p2}(z)\right]}_{\approx 0}\right.$$

$$\left. + R(z)\left[\underbrace{\frac{G_p(z)}{\hat{G}_p(z)}}_{\approx 1} + G_cG_p(z)\right]\right\} \Rightarrow \quad Y(z) \cong R(z)$$

Example



Develop a DID for the system shown.

Solution

To reject the disturbance, the following relationship has to be satisfied:

$$D(z)Z\left\{G_{zoh}(s)\frac{1}{Js}\right\} = D(z)G_d(z)\underbrace{Z\left\{G_{zoh}(s)\frac{K_a}{Js}\right\}}_{\triangleq G_p(z)}$$

Hence

$$G_d(z) = \frac{\frac{1}{J}(1-z^{-1})Z\left\{\frac{1}{s^2}\right\}}{\frac{K_a}{J}(1-z^{-1})Z\left\{\frac{1}{s^2}\right\}} = \frac{1}{K_a}$$

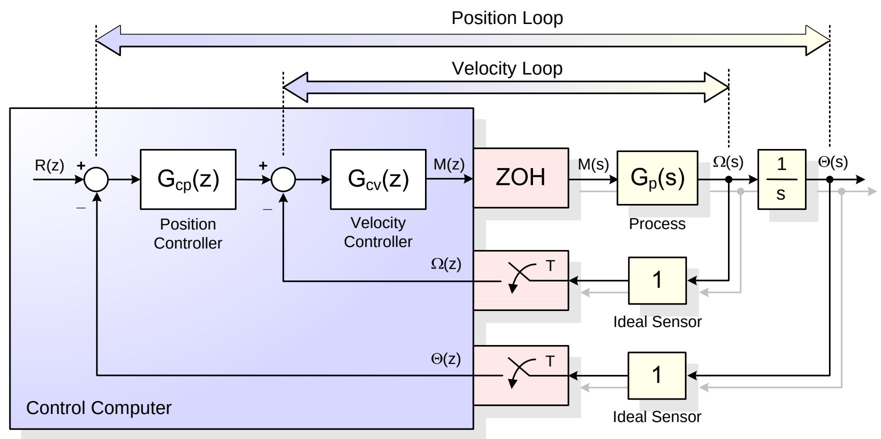
Summary

- CFC is used to improve both the *tracking accuracy* and the speed of response.
  - Very sensitive to errors in parameter estimates as well as changing physical parameters
  - No effect on the disturbance rejection
- DID is employed when disturbance can be measured.
  - With excellent estimates on physical parameters, DID can totally eliminate the adverse effects of the disturbance.
- None of these controllers is sufficient to control a plant in real-world. A feedback controller is needed to account for *non-ideal* conditions:
  - Changing physical parameters
  - Disturbances including noise
  - Unmodelled system dynamics

Cascade Control

- Many industrial motion control applications contain cascaded control loops:
  - *Fast* inner velocity loop
  - *Slow* outer position control loop feed velocity commands to the inner loop.
- First velocity controller is designed.
- With the velocity loop in place, a position controller is developed.

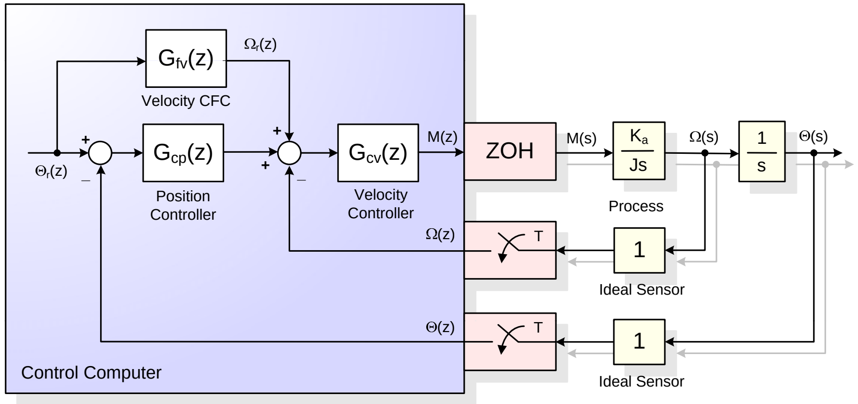
Cascade Control (Cont'd)



Industrial Motion Controller

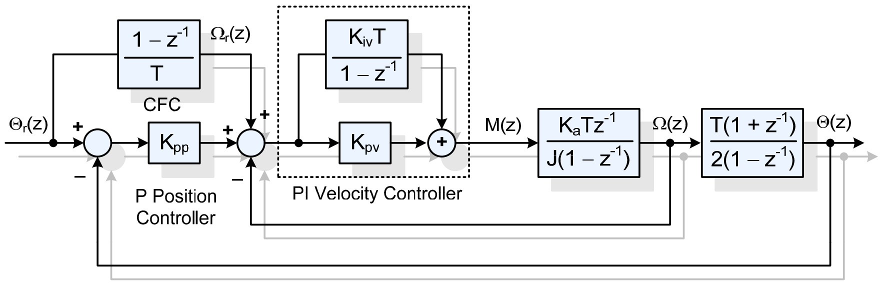
- Cascaded motion controllers are common in industry:
  - Factory automation
  - Robotics
  - CNC machine tools
- (Defacto) standard motion controllers employ a PI velocity controller (inner loop) along with a P position controller (outer loop).
  - A velocity CFC improves the command tracking accuracy of the controlled system.

Example



Design an industry-standard motion controller for the system shown.

Solution



The overall controller system (in z-domain) is as illustrated. Please refer to the last example of Chapter 2a for process model development. Here, a simple velocity CFC is utilized:

$$\omega_r(k) = \underbrace{\frac{\theta_r(k)-\theta_r(k-1)}{T}}_{average\ velocity} \Rightarrow \frac{\Omega_r(z)}{\Theta_r(z)} = \frac{1-z^{-1}}{T}$$

Solution (Cont'd)

Now it is time to design the controllers (i.e. determine their gains).

Velocity Loop:

$$G_{OL}(z) = \left(\frac{K_aT}{J}\right)\frac{(K_{iv}T+K_{pv})z-K_{pv}}{(z-1)^2} = K_v\frac{z-b_v}{(z-1)^2}$$

where

$$K_v \triangleq \frac{K_aT}{J}(K_{iv}T+K_{pv})$$

$$b_v \triangleq \frac{K_{pv}}{K_{iv}T+K_{pv}}$$

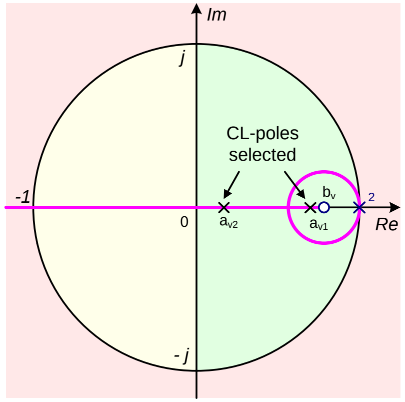
Solution (Cont'd)

Using root locus techniques, one can place the CL poles to the desired locations inside the unit circle considering the technical specs. Hence, the CLTF of the velocity loop becomes

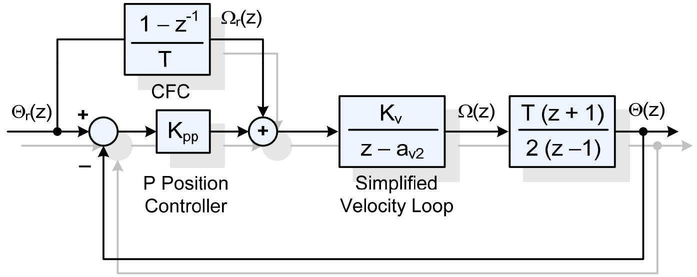
$$\frac{\Omega(z)}{\Omega_r(z)} = G_{VL}(z) = K_v \frac{z - b_v}{(z - a_{v1})(z - a_{v2})}$$

Since $a_{v1} \approx b_v$, the effect of the slow pole is almost cancelled by the zero. Hence, the fastest CL pole @ a_{v2} will dominate the dynamics of the loop:

$$G_{VL}(z) \cong \frac{K_v}{z - a_{v2}}$$



Solution (Cont'd)



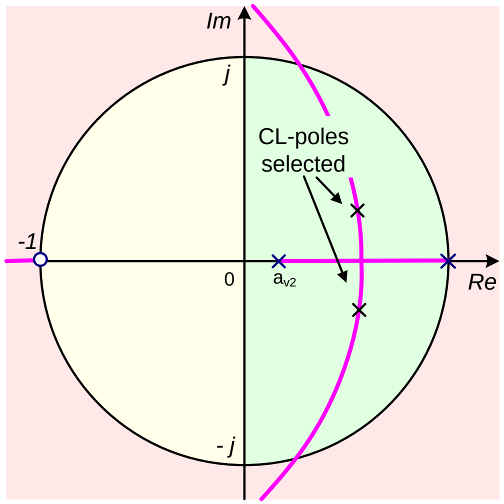
Position Loop:

With the simplified velocity loop in place, the OL-TF of the position loop becomes

$$G_{OL}(z) = \frac{K_p (z+1)}{(z - a_{v2})(z - 1)} \quad \text{where} \quad K_p \triangleq \frac{K_{pp} K_v T}{2}$$

Solution (Cont'd)

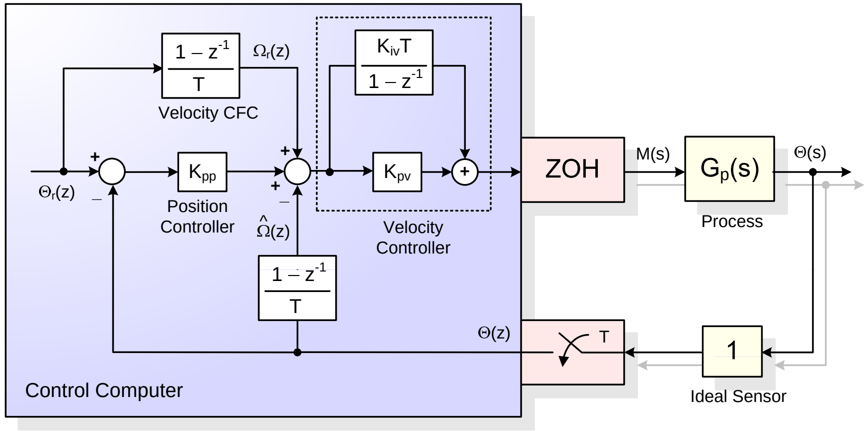
Using root locus techniques, the CL poles of the position loop are placed inside the unit circle to satisfy the constraints set by the technical specs.



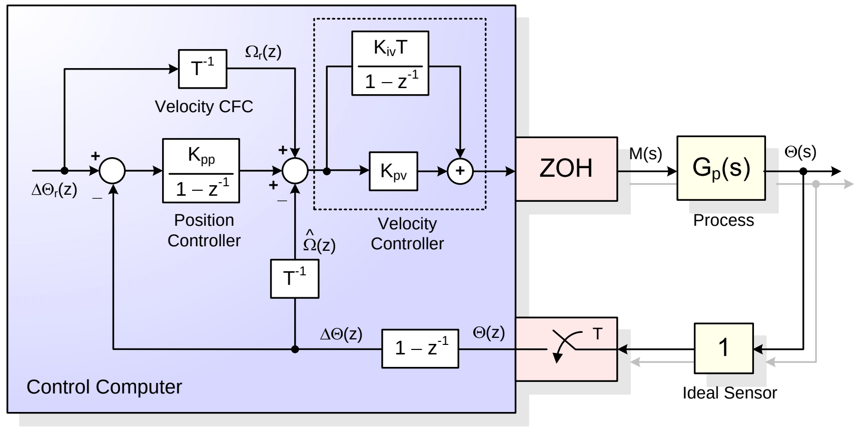
Motion Controllers with only Position Feedback

- In most motion control systems, only position is measured via optical position encoder.
- Employing a second sensor to measure velocity is *not* a viable option:
  - Higher cost
  - Less reliability
- Thus, velocity is to be estimated via first-order difference technique with position measurements at hand.
- There exist two different versions of such control system:
  - Absolute position
  - Incremental position

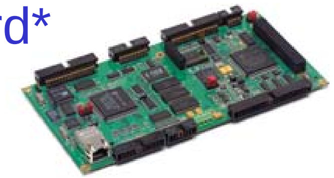
MCS with Absolute Position

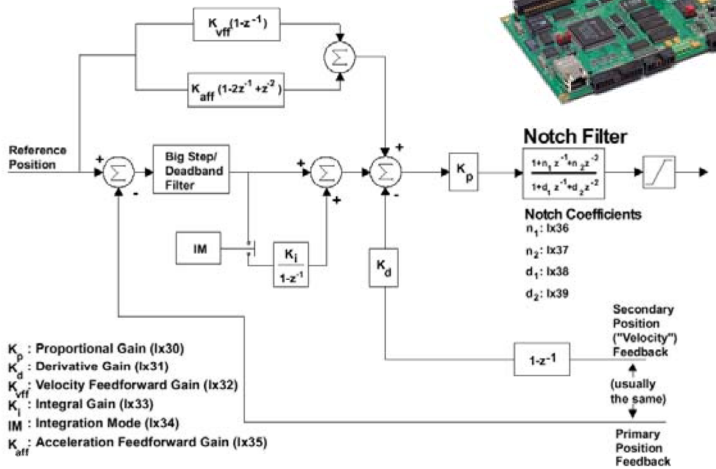


MCS with Incremental Position



PID Algorithm of PMAC2 Motion Control Card*





 [*] Courtesy of Delta-Tau Corp.